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THE VELOCITY OF SHEAR FRACTURE

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ABSTRACT

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The maximum velocity of fracture in an homogeneous isotropic elastic medium under pure shear stress and with an added compressive stress is computed by an extension of Yoffe's method. The maximum velocity of shear fracture is smaller than the velocity of transverse waves in the medium. It increases as Poisson's ratio increases and decreases as the compressive stress increases. The maximum velocity of pure shear fracture is higher than the velocity of pure tensile fracture in the same medium. For a medium with Poisson's ratio of 0.25 the former is 0.775 and the latter is 0.631 of the transverse wave velocity.

AUTHOR

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INTRODUCTION

The geographical distribution of the sense of first motions from earthquakes strongly suggests that a sudden shearing motion takes place at the earthquake focus. The actual mechanism at the focus is not known, but the shearing motion could possibly be caused by rupture of two types, a) by the reopening by relative motion of two sides of a fused fault or b) by the shear fracture (particle motion parallel to fracture surface and normal to fracture front) of an unbroken medium. In either case the rupture would travel at a finite velocity. Because of uncertainties about the materials and the nature of fusion, theoretical determination of the velocity of reopening of fused fault is difficult. The determination of the velocity of shear fracture of an unbroken medium is more tractable. Press, Ben Menahem and Toksöz (1961) and Ben-Menahem and Toksöz (1962) have computed a fault length and rupture velocity by two different methods from an analysis of surface waves. In this paper we confine ourselves to studying theoretically the stress distribution at the head of a moving shear fracture, with and without superimposed compression, and to see how the stress distribution sets an upper limit to the velocity of shear fracture propagation. We then obtain the maximum velocity of shear fracture under various conditions.

The theoretical maximum velocity of propagation of tensile fractures was determined by Yoffe (1951). She found that a fracture loses energy by branching after attaining a velocity equal to 0.6 of the transverse wave velocity. A more general analysis has been given by Bilby and Bullough (1954), who derived the expressions for the stress distribution around the head of a fracture propagating in an elastic medium under an applied stress P_{ij}. Craggs (1960) has treated the problem of a growing semi-infinite fracture in which an internal stress follows the fracture front. McClintock and Sukhatme (1960), Barenblatt and Cherepanov (1961), and Field and Baker (1962) have studied the problem of longitudinal shear fracture.

STRESS DISTRIBUTION

The propagating fracture has been treated by Yoffe (1951) and by Bilby and Bullough (1954) as a disturbance of constant length that moves across an elastic medium. If attention is concentrated on regions in the immediate vicinity of the fracture front, the effect of the tail end of the stress distribution near the front will be small. Referring to Figure 1, the fracture front extends from $-\infty$ to ∞ along the z axis. The fracture propagates in the x direction with a velocity of c. The Y axis is normal to the fracture surface. The length of the fracture is 2a. The fracture surface is assumed to be plane and frictionless. Following the notation of Bilby and Bullough (1954) P_{ij} is defined as the applied stress tensor, and p_{ij} as the stress around the fracture front which we seek to determine. If the applied stress P_{ij} is removed then

$$p_{ij} = 0 \quad \text{at} \quad \infty \tag{1}$$

and for $-a \leq x' \leq a$ where $x' = x - ct$

Problem I $$p_{xy} = -P_{xy} = -S \tag{2}$$

Problem II $$p_{yy} = -P_{yy} = -T \tag{3}$$

Problem III $$p_{yz} = -P_{yz} = -S_1 \tag{4}$$

We are primarily interested only in Problem I (pure shear fracture) and in Problem II (tensile fracture). By symmetry we may confine ourselves to the half-space $y > 0$. The boundary conditions for $y=0$ in this half space are given by

Problem I Shear stress

$$\begin{aligned} p_{xy} &= -S \quad \text{for } -a \leq x' \leq a \\ p_{yy} &= 0 \quad \text{for all } x' \\ u &= 0 \quad \text{for } |x'| > a \end{aligned} \tag{5}$$

Problem II Tensile stress

$$\begin{aligned} p_{yy} &= -T \quad \text{for } -a \leq x' \leq a \\ p_{xy} &= 0 \quad \text{for all } x' \\ v &= 0 \quad \text{for } |x'| > a \end{aligned} \tag{6}$$

Where u and v are the displacements in the x and y directions respectively.

Let a polar coordinate system (r, θ) be defined such that $x' - a = r \cos\theta$, $y = r \sin\theta$. Then p_{ij} can be obtained for a small constant r ($r/a \ll 1$) and different θ's, in the neighborhood of the fracture front. Part of the solution obtained by Yoffe (1951) and Bilby and Bullough (1954) is given below. The shear stress at the fracture front is given by

Problem I

$$-p_{xy} = S \cdot \frac{1}{H}\left[-2\gamma \frac{\cos(\theta_1/2)}{\sqrt{2k_1}} + \frac{(1+\beta^2)^2}{2\beta} \frac{\cos(\theta_2/2)}{\sqrt{2k_2}}\right] \qquad (7)$$

Problem II

$$p_{xy} = T \cdot \frac{1}{N}\left[\frac{\sin(\theta_1/2)}{\sqrt{2k_1}} - \frac{\sin(\theta_2/2)}{\sqrt{2k_2}}\right] \qquad (8)$$

where

c = fracture velocity

c_1 = dilatation wave velocity

c_2 = transverse wave velocity

$\gamma = (1 - c^2/c_1^2)^{\frac{1}{2}}$

$\beta = (1 - c^2/c_2^2)^{\frac{1}{2}}$

$N = -(1 + \beta^2)/\gamma + 4\beta/(1 + \beta^2)$

$H = -2\gamma + (1 + \beta^2)/2\beta^2$

$\theta_1 = \tan^{-1}(\gamma \tan\theta)$

$\theta_2 = \tan^{-1}(\beta \tan\theta)$

k_0 = small arbitrary constant

$$k_1 = k_o \quad \left[1 - (c^2/c_1^2)\sin^2\theta\right]^{\frac{1}{2}}$$

$$k = k_o \quad \left[1 - (c^2/c_2^2)\sin^2\theta\right]^{\frac{1}{2}}$$

In the case of a shear fracture in a medium under compression, we assume that the compressive stress is applied normal to the fractrue surface. While physically it may be difficult to imagine a purely compressive fracture, mathematically it is possible to calculate the stress p_{ij} in such a case by simply inverting the sign of T in Problem II. The stresses p_{ij} could then be interpreted as due to the imposition of the compressive stress on a medium in which a fracture is already propagating because of a different applied stress. In our case, the compressive stress -T is applied to a medium in which a shear stress S is already causing a shear fracture to propagate. Therefore, for a combination of Problems I and II the shear stress in the neighborhood of the fracture front is, by superposition,

$$p_{xy} = S\left[\frac{1}{H}\left(-2r\frac{\cos(\theta_1/2)}{\sqrt{2k_1}} + \frac{(1+\beta^2)^2}{2\beta}\frac{\cos(\theta_2/2)}{\sqrt{2k_2}}\right) + \frac{T}{S}\;\frac{1}{N}\left(\frac{\sin(\theta_1/2)}{\sqrt{2k_1}} - \frac{\sin(\theta_2/2)}{\sqrt{2k_2}}\right)\right] \quad \ldots\ldots (9)$$

As we are primarily interested in shear fractures -T/S is always ≤ 1. The well known transformation relations (Durelli et. al. (1958) are then used to obtain p_{rr}, $p_{\theta\theta}$, $p_{r\theta}$ in polar coordinates from p_{xx}, p_{yy}, p_{xy}.

RESULTS

The independent variables in the expressions (7), (8) and (9) are the Poisson's ratio σ (which determines the ratio c_1/c_2). the relative fracture velocity c/c_2, the ratio of compressive to shear stress $-T/S$, and the angle θ. The solutions p_{ij} for all three expressions were computed on the Rice University computer. One solution of Problem II served as a check by making it possible to compare our result with that of Yoffe (1951). For purely tensile fractures in a medium of Poisson's ratio 0.25, the maximum fracture velocity is given by Yoffe to be $0.6c_2$ as compared to our result of $0.631c_2$. The difference is due to the greater precision of our computation. For a medium of Poisson's ratio 0.25, our values of C_b for pure tensile and pure shear fractures also agree with those obtained by Craggs (1960).

The variables in the expressions (7), (8), and (9) were varied in the following ranges: σ from 0.10 to 0.30, c/c_2 from 0.20 to 1.00, $-T/S$ from 0.00 to 1.00, θ from 0° to 90°. A typical set of curves is presented in Fig. 2 for $\sigma = 0.25$ and $-T/S = 0$. Fig. 3 shows the set of curves for $\sigma = 0.25$ and $-T/S = 0.6$. Since we are dealing with shear fractures it is reasonable to assume that the fracture will propagate in the direction in which the shear stress $p_{r\theta}$ is maximum.. For low values of c/c_2 the maximum occurs at $\theta = 0^\circ$, and the fracture propagates in a straight line. With increase of velocity the relative amplitude of the maximum decreases until at a certain value of c/c_2, $p_{r\theta}$ shows no variation over a wide range of θ around $\theta = 0^\circ$. At this velocity the no preferred direction of propagation. Therefore it is considered that the fracture will form branches. The available energy of the fracture formation will then be divided and the fracture will slow down to a value lower than the critical branching velocity. This process will be repeated for each of the branches. Thus the fracture can never exceed the critical branching velocity c_b. Fig. 4 shows the variation of $p_{r\theta}$ with θ, at the critical branching velocity, for $\sigma = 0.25$ and for different -T/S. Values of the critical branching velocity, which is the maximum velocity of shear fracture, are given in Table 1 for different σ and -T/S ratios. One notices that the branching velocity increases as Poisson's ratio increases and as the compressive stress decreases. For comparison the branching velocity for purely tensile fractures if given in Table 2. It can be seen that for the same medium the branching velocity for a pure shear fracture is higher than a pure tensile fracture.

DISCUSSION

Some of the assumptions used in obtaining values for the maximum velocity of shear fractures are far removed from reality. A moving fracture can maintain a constant length only if the medium heals completely after the tail end of the fracture. This is obviously physically impossible. However, the results obtained should still be valid as we are primarily concerned with the stress in the immediate vicinity of the fracture front, which is far removed from the tail end. Secondly plane frictionless fracture surfaces do not exist in nature. Any shear movement along a real fracture will bring opposing frictional forces into play. It appears however, that the opposing frictional forces will only affect the magnitude of S, the applied shear stress. From expression (7) it is apparent that this will not affect the stress distribution. The maximum fracture velocity of pure shear fracture is therefore unaffected by frictional forces. Our results appear to be confirmed for shear fractures in the earth. In the case of the Chilean earthquake of 1960 Press, et al (1960) have found a rupture velocity near the velocity of transverse waves in crustal rocks. Similar results were obtained by Ben Menahem and Toksöz (1962) in the case of the Mongolian earthquake of 1957.

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TABLE 1. Values of maximum velocity for shear fractures

$-T/S$	0.00	0.20	0.40	0.60	1.00	
σ						
0.10	0.721	0.668	0.650	0.640	0.625	0.621
0.15	0.738	0.685	0.667	0.658	0.652	0.649
0.20	0.756	0.703	0.686	0.676	0.671	0.667
0.25	0.775	0.722	0.704	0.695	0.690	[illegible]
0.30	0.794	0.741	0.724	0.715	0.709	0.70[illegible]

TABLE 2. Values of maximum velocity for pure tensile fractures

σ	0.10	0.15	0.20	0.25	0.30
c_b	0.561	0.583	0.606	0.631	0.657

Fig. 1 A fracture of constant length 2a is moving in the X direction in an elastic medium. AB is part of the fracture front which extends from $-\infty$ to ∞ in this Z direction.

Figure 2 Stress distribution near the front of a pure shear fracture ($-T/S = 0$) for different velocities. The fracture is moving in an elastic medium with a Poisson's ratio of 0.[illegible].

Figure 3 Stress distribution near the front of a shear fracture for different velocities. The compressive to shear stress ratio ($-T/S$) is 0.6. The fracture is moving in an elastic medium with a Poisson's ratio of 0.25.

Figure 4 Shear stress $p_{r\theta}$ distribution at the front of shear fracture at branching velocity, for different values of compressive shear stress ratio. The Poisson's ratio of the medium is 0.25.

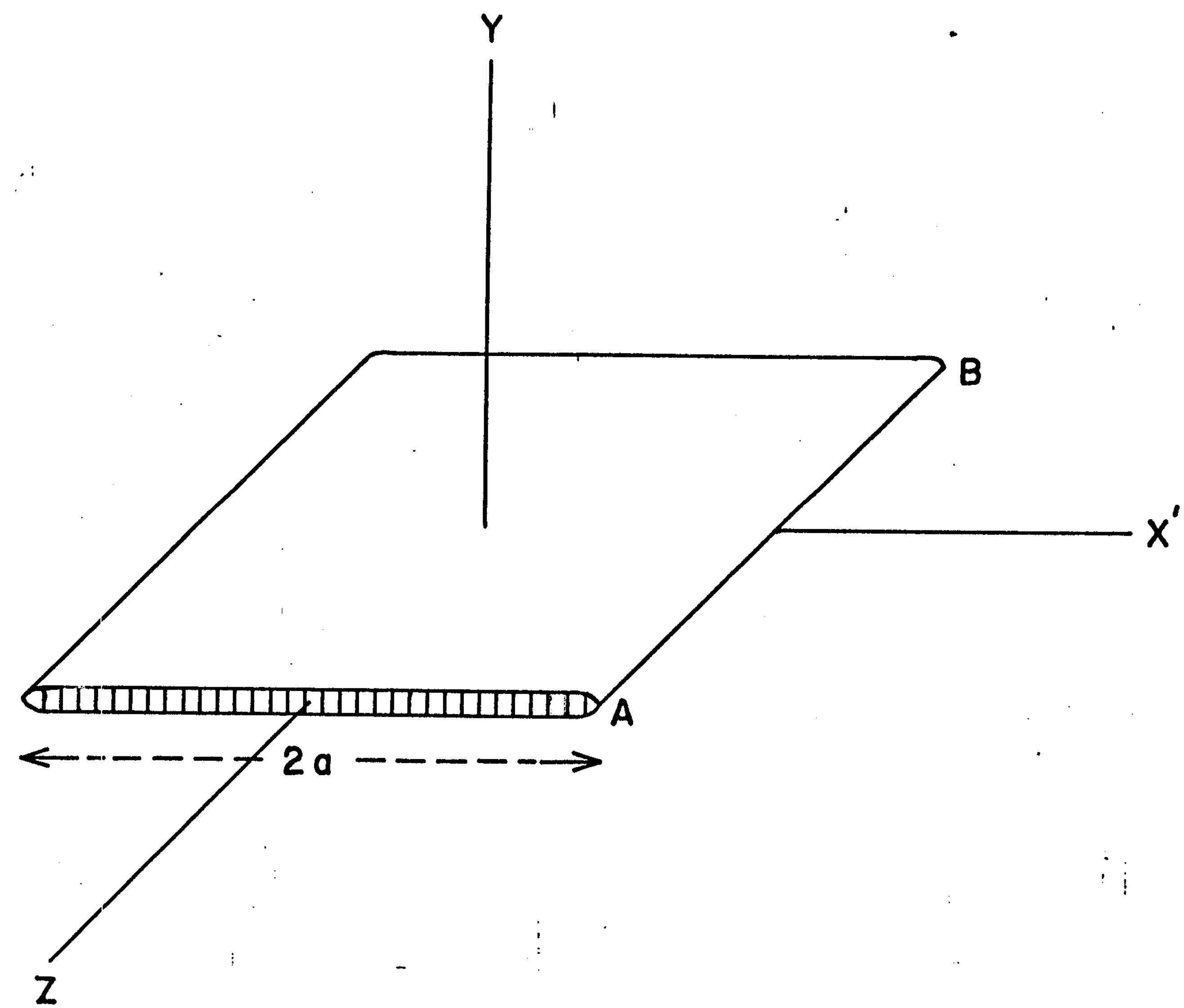
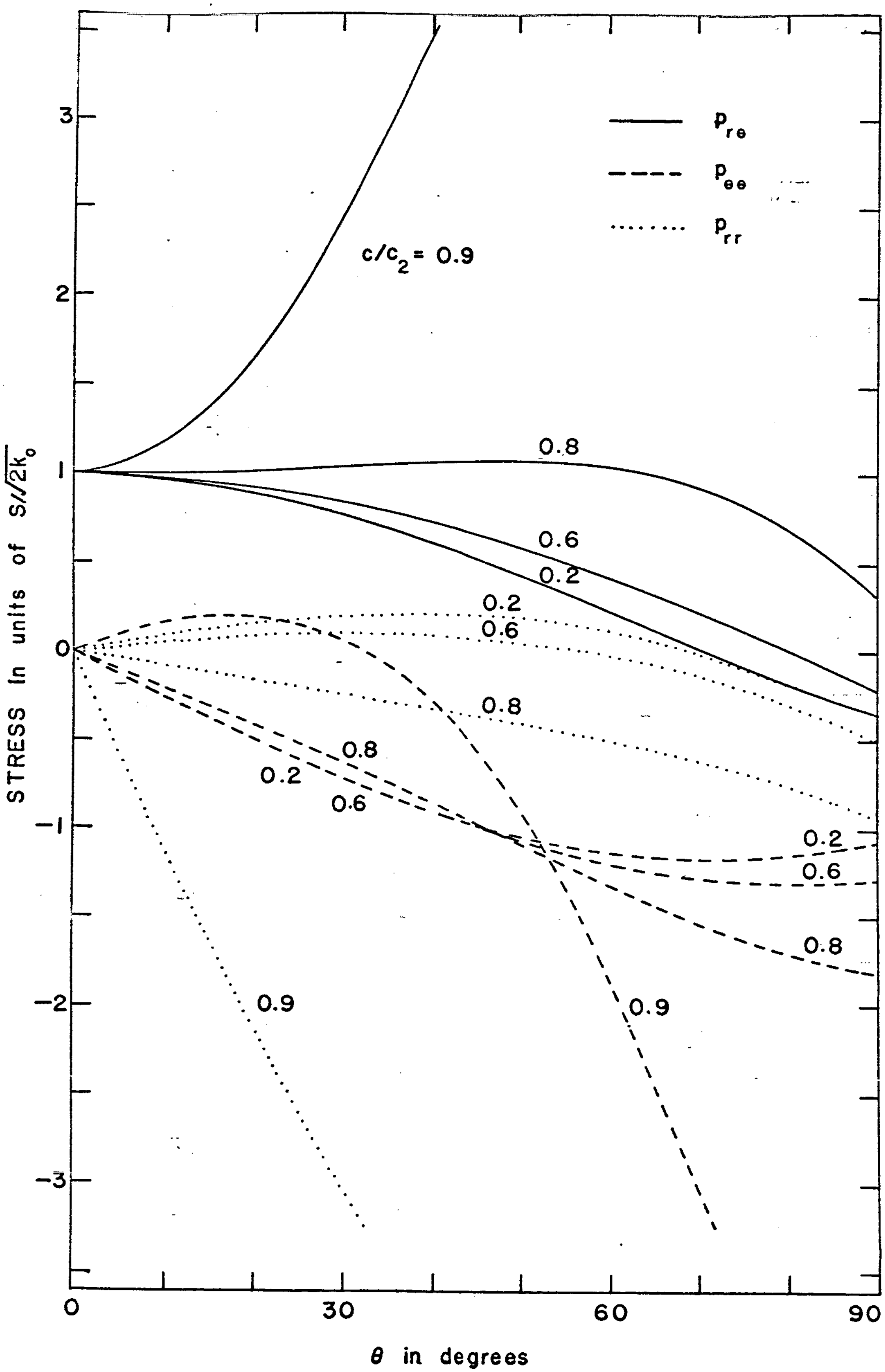
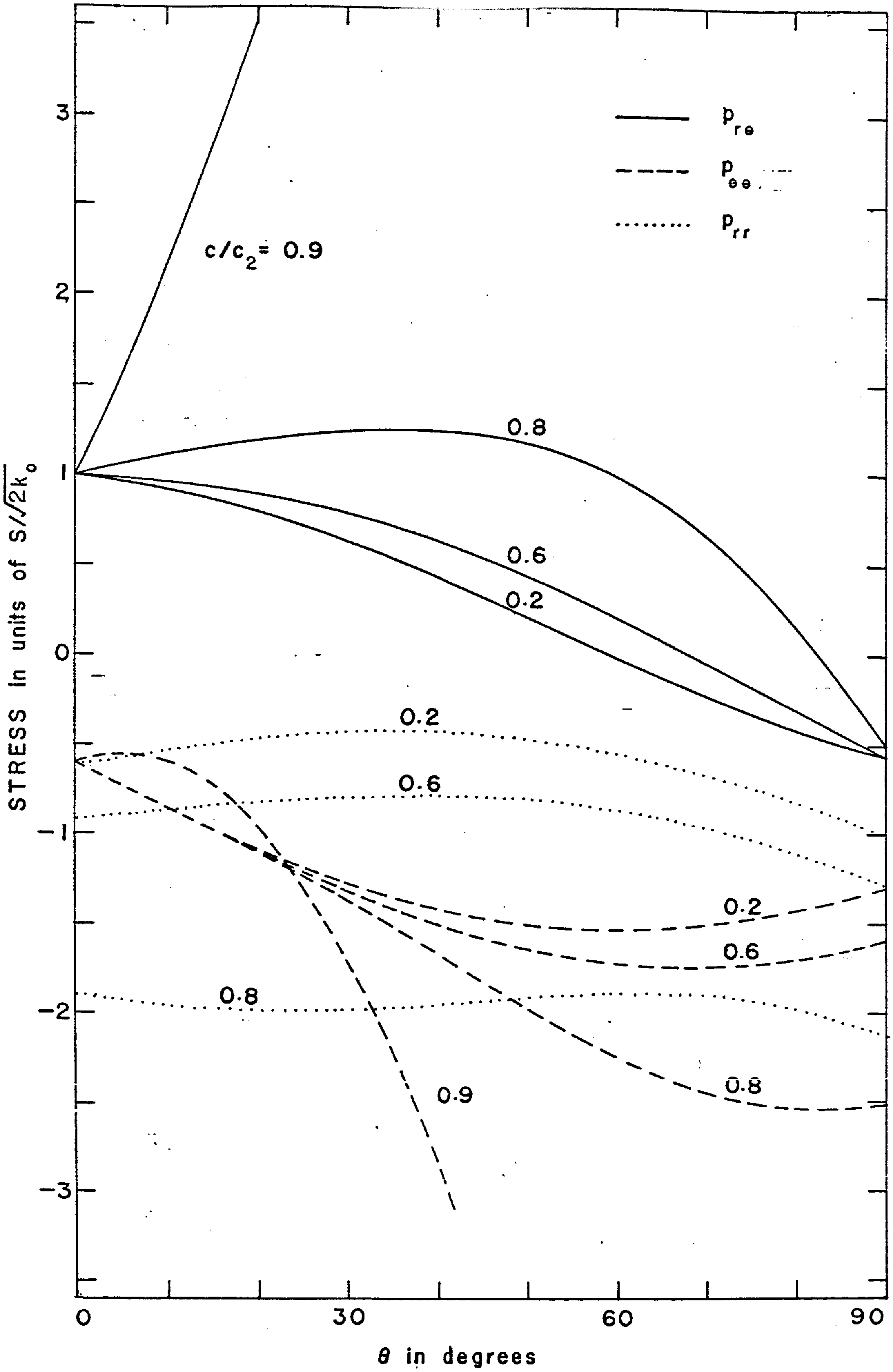


fig 2



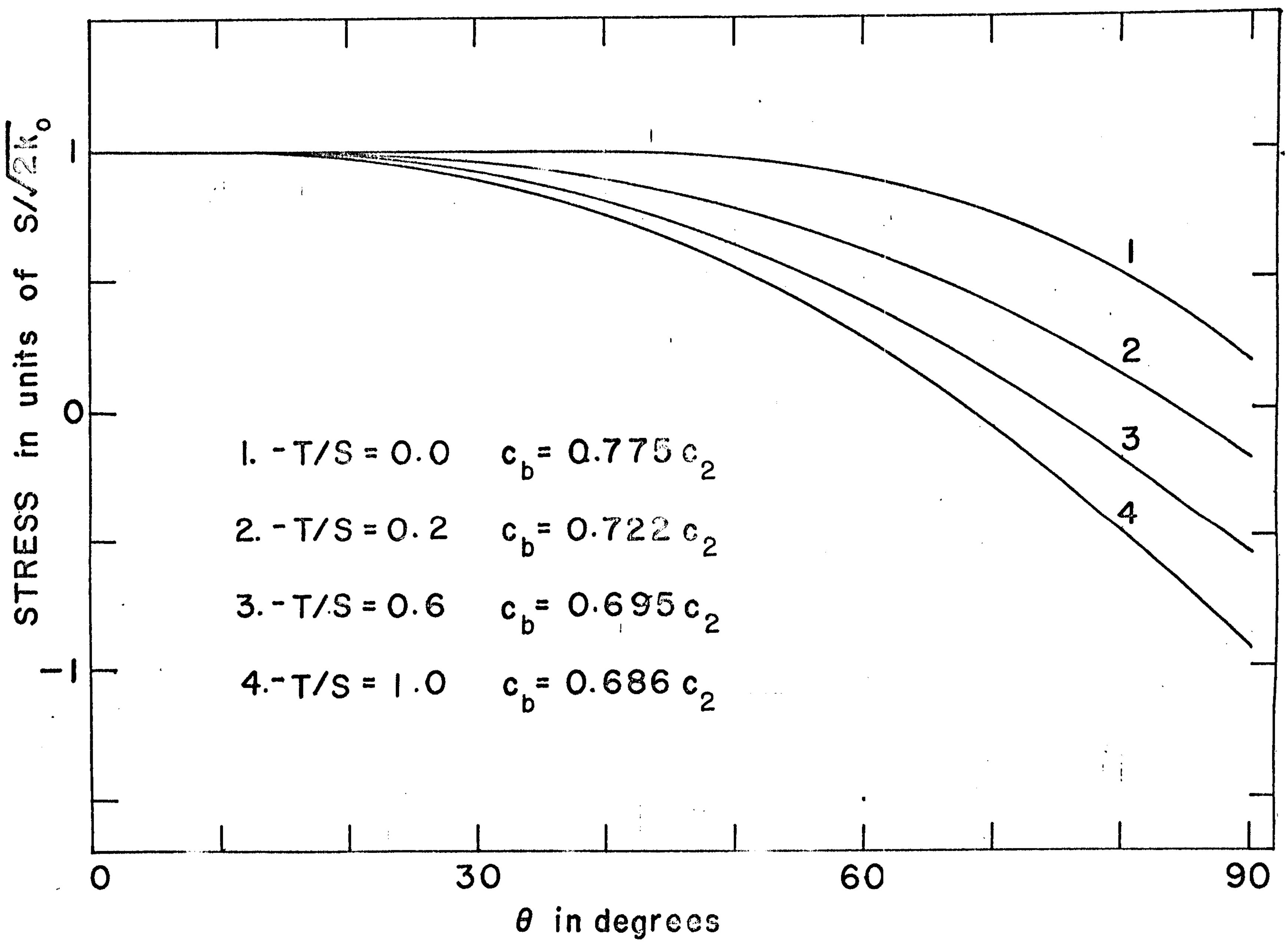
$p_{r\theta}$
$p_{\theta\theta}$
p_{rr}
$c/c_2 = 0.9$
0.8
0.6
0.2
0.2
0.6
0.8
0.8
0.2
0.6
0.2
0.6
0.8
0.9
0.9
3
2
1
0
−1
−2
−3
STRESS in units of $S/\sqrt{2}k_0$
0
30
60
90
θ in degrees

Fig 2



p_rθ
p_θθ
p_rr
c/c₂ = 0.9
0.8
0.6
0.2
0.2
0.6
0.2
0.6
0.8
0.8
0.9
3
2
1
0
−1
−2
−3
STRESS in units of S/√2k₀
0
30
60
90
θ in degrees

fig 3


STRESS in units of $S/\sqrt{2k_o}$
1
0
-1
0
30
60
90
θ in degrees
1. -T/S = 0.0 $c_b = 0.775\,c_2$
2. -T/S = 0.2 $c_b = 0.722\,c_2$
3. -T/S = 0.6 $c_b = 0.695\,c_2$
4. -T/S = 1.0 $c_b = 0.686\,c_2$
1
2
3
4

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